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LETTER TO THE EDITOR

A new class of completely integrable quantum spin chains

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Abstract. A large (infinitely dimensional) class of completely integrable (possibly non-autonomous) spin-$\frac{1}{2}$ chains is discovered associated to an infinite-dimensional Lie algebra of infinite rank. The complete set of integrals of motion is constructed explicitly, as well as their eigenstates and spectra. As an example we outline the *kicked Ising model*: Ising chain periodically kicked with transversal magnetic field.

During the past three decades intricate algebraic techniques (under the names *quantum inverse scattering* or *algebraic Bethe ansatz*) have been developed [1] in order to construct integrable quantum many-body (IQM) dynamical systems and the associated complete sets of integrals of motion. Integrability of a quantum many-body dynamical system is defined in a generalized Liouvilean sense; namely by the existence of an infinite set of (independent and local) conservation laws. All of the IQM systems discovered to date are one-dimensional, typically SU(2) spin chains or related systems. Quantum integrability is *non-generic* but of great importance, since it has been shown recently [2] that the existence of non-trivial conservation laws generically leads to ideal transport properties with infinite Kubo transport coefficients, and deviation from *quantum ergodicity* in general.

In this letter we present a new and elementary approach to the construction of IQM one-dimensional lattice systems. It is based on the particular infinite-dimensional dynamical Lie algebra (DLA) generated and represented by the essential dynamical observables (in our case it is generated by the Ising Hamiltonian $\sum_j \sigma^x_j \sigma^x_{j+1}$ and the interaction with the transversal external field $\sum_j \sigma^z_j$) and for which the 'transfer matrix' can be explicitly constructed from the *commutativity condition*. We show that *any* element H of DLA may be considered as a Hamiltonian of an IQM system and construct an analytic DLA-valued function $T(\boldsymbol{\lambda})$ of a possibly vectorial spectral parameter $\boldsymbol{\lambda} \in \mathbb{C}^N$ (for some $N \geqslant 1$), commuting with H, $[H, T(\boldsymbol{\lambda})] \equiv 0$. $T(\boldsymbol{\lambda})$ is a formal analogue of the *logarithm of the transfer matrix*. The integrals of motion (conserved charges and currents) are derived as coefficients of Taylor expansion of $T(\boldsymbol{\lambda})$ around $\boldsymbol{\lambda} = 0$. Therefore we have an infinite-dimensional class of IQM Hamiltonian systems. Furthermore, *real* DLA of self-adjoint observables generates infinite-dimensional unitary dynamical Lie group of even larger class of integrable quantum many-body propagators of possibly non-Hamiltonian (non-autonomous, for example, *periodically kicked*) IQM systems. As an example we calculate a kicked one-dimensional Ising chain periodically kicked with a transversal external field. Moreover, we explicitly calculate the complete set of eigenstates and spectra of the conserved charges (including the Hamiltonian).

Let us consider infinite chains of spins having the magnitude $\frac{1}{2}$ on each site j. A spin at site j is described by spin-$\frac{1}{2}$ variables (Pauli matrices) σ^s_j, $s \in \{1 = x, 2 = y, 3 = z\}$, obeying the standard commutation relations $[\sigma_j^p, \sigma_k^r] = 2\delta_{jk}\sigma_j^p\sigma_k^r = 2\mathrm{i}\delta_{jk}\sum_s \epsilon_{prs}\sigma_j^s$, and a unit matrix $\sigma_j^0 = 1$. We start with the following Lie algebra $\mathfrak{U}$ over an infinite spin chain spanned by the spatially homogeneous local observables

$$Z_{[s_1 s_2 \ldots s_p]} = \sum_{j=-\infty}^{\infty} \sigma_j^{s_1}\sigma_{j+1}^{s_2}\ldots\sigma_{j+p-1}^{s_p}. \tag{1}$$

We assume that $s_1, s_p \neq 0$, and that we have infinite direct products of unit matrices σ_j^0 to the left- and right-hand side of each term in (1).

The *order* of the local observable A is defined as the maximal number of digits p of some observable (1) in the expansion of A in terms of basis (1). We are interested in non-trivial infinite-dimensional subalgebras of $\mathfrak{U}$ for which the number of elements with order smaller than p grows algebraically (as a function of p) and not exponentially ($\sim 4^p$) as for $\mathfrak{U}$†. Indeed we found subalgebra $\mathfrak{S}$, which we call DLA (essentially generated by $Z_{[3]}$ and $Z_{[11]}$), and spanned by two infinite sequences of self-adjoint observables U_n and V_n,

$$\begin{aligned} U_n &= \begin{cases} Z_{[1(3^{n-1})1]} & n \geqslant 1 \\ -Z_{[3]} & n = 0 \\ Z_{[2(3^{-n-1})2]} & n \leqslant -1 \end{cases} \\ V_n &= \begin{cases} Z_{[1(3^{n-1})2]} & n \geqslant 1 \\ Z_{[0]} & n = 0 \\ -Z_{[2(3^{-n-1})1]} & n \leqslant -1 \end{cases} \end{aligned} \tag{2}$$

for $-\infty < n < \infty$ ((3^n) indicates digit 3 being repeated n times), which satisfy the following commutation relations

$$\begin{aligned} &[U_m, U_n] = 2\mathrm{i}(V_{m-n} - V_{n-m}) \\ &[V_m, V_n] = 0 \\ &[U_m, V_n] = 2\mathrm{i}(U_{m+n} - U_{m-n}). \end{aligned} \tag{3}$$

The order of observables U_n and V_n is $|n| + 1$. The covering algebra $\mathfrak{U}$ is equiped with the Euclidean metric associated with the bilinear form (scalar product)

$$(A|B) = \lim_{L\to\infty} \frac{1}{L2^L} \mathrm{tr}_L(A^\dagger B) \tag{4}$$

(tr_L is a trace for a finite system of size L) with respect to which (1) is an orthonormal (ON) basis. Further, U_n and V_n form an ON basis of DLA $\mathfrak{S}$ in the same metric. Note that (4) is *invariant* with respect to the *adjoint map*, $(\mathrm{ad}\,A)B = [A, B]$, namely $((\mathrm{ad}\,A^\dagger)B|C) = (B|(\mathrm{ad}\,A)C)$.

Conservation laws in general autonomous case

Let us assume that the Hamiltonian H and the logarithm of the transfer matrix T belong to DLA $\mathfrak{S}$. We write

$$H = \sum_{m=-m_-}^{m_+} (h_m U_m + g_m V_m) \tag{5}$$

† In the forthcoming publication (T Prosen 1998 Quantum invariants of motion in a generic many-body system *Preprint* cond-mat/9803358) we discuss an ‘exponentially large’ *invariant dynamical Lie subalgebra of* $\mathfrak{U}$, corresponding to a *non-integrable* kicked Heisenberg XXZ chain, whose power grows as $\sim 1.7^p$ and in which we find (few) non-trivial local conservation laws that explain deviations from quantum ergodicity and normal transport as observed numerically in [3].

$$T = \sum_{m=-\infty}^{\infty} (a_m U_m + b_m V_m) \tag{6}$$

where the Hamiltonian H has a finite order $M := \max\{m_+, m_-\}$. The commutativity relation $[H, T] = 0$ results in the system of difference equations

$$\begin{aligned} &\sum_m h_m (a_{-n+m} - a_{n+m}) = 0 \\ &\sum_m [h_m (b_{n-m} - b_{-n+m}) + g_m (a_{n+m} - a_{n-m})] = 0 \end{aligned} \tag{7}$$

which can be solved with an ansatz

$$\begin{aligned} a_n &= a_+ \lambda^n \qquad & b_n &= b_+ \lambda^n \\ a_{-n} &= a_- \lambda^n \qquad & b_{-n} &= b_- \lambda^n \end{aligned} \tag{8}$$

for $n \geqslant 0$. Quite surprisingly, the resulting homogeneous system

$$\begin{pmatrix} h(\lambda) & -h(\lambda^{-1}) & 0 & 0 \\ g_a(\lambda) & 0 & h(\lambda^{-1}) & -h(\lambda^{-1}) \\ 0 & g_a(\lambda) & h(\lambda) & -h(\lambda) \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \\ b_+ \\ b_- \end{pmatrix} = 0 \tag{9}$$

has rank 2 for *any* value of the *spectral parameter* λ, where $h(\lambda)$ and $g_a(\lambda) := g(\lambda) - g(\lambda^{-1})$ are the polynomials

$$h(\lambda) = \sum_{m=-m_-}^{m_+} h_m \lambda^m \qquad g(\lambda) = \sum_{m=-m_-}^{m_+} g_m \lambda^m.$$

Hence, there are two linearly independent solutions of (9) (up to an arbitrary common prefactor), namely

$$\begin{aligned} a_+(\lambda) &= h(\lambda^{-1}) \qquad & b_+(\lambda) &= g(\lambda^{-1}) \\ a_-(\lambda) &= h(\lambda) \qquad & b_-(\lambda) &= g(\lambda) \end{aligned} \tag{10}$$

and

$$a_+(\lambda) = a_-(\lambda) \equiv 0 \qquad b_+(\lambda) = b_-(\lambda) \equiv 1. \tag{11}$$

(i) First let us consider the case where the solutions $a_\pm(\lambda), b_\pm(\lambda)$ are given by (10). The global uniform solution (for all $n \in \mathbb{Z}$) is given by a linear combination of $N := m_+ + m_- + 1$ solutions (10)

$$a_n = \sum_{m=1}^{N} c_m a_+(\lambda_m) \lambda_m^n \qquad b_n = \sum_{m=1}^{N} c_m b_+(\lambda_m) \lambda_m^n \tag{12}$$

for $n \geqslant 0$, and

$$a_n = \sum_{m=1}^{N} c_m a_-(\lambda_m) \lambda_m^{-n} \qquad b_n = \sum_{m=1}^{N} c_m b_-(\lambda_m) \lambda_m^{-n} \tag{13}$$

for $n \leqslant 0$. N-tuple of *spectral parameters* $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)$ is an arbitrary subset of a complex unit disk $|\lambda_m| < 1$ (in order to ensure convergence of T) while the coefficients c_m are determined by gluing the solutions (12) and (13) on $m_+ + m_- = N - 1$ sites around $n = 0$, giving a homogeneous system of $N - 1$ linear equations

$$\sum_{m=1}^{N} (\lambda_m^n - \lambda_m^{-n}) c_m = 0 \qquad n = 1, \ldots, N \tag{14}$$

with a general (polynomial) solution

$$c_m(\boldsymbol{\lambda}) = (-1)^m \lambda_m^{N-1} \prod_{j\leqslant k}^{j,k\neq m} (1-\lambda_j\lambda_k) \prod_{j<k}^{j,k\neq m} (\lambda_j - \lambda_k). \tag{15}$$

Logarithmic transfer matrix $T(\boldsymbol{\lambda})$ is a holomorphic function in $\boldsymbol{\lambda}$, and the coefficients of its Taylor expansion around $\boldsymbol{\lambda} = 0$ also commute with H. After some simple series manipulations we easily find an infinite sequence of independent integrals of motion, namely the *conserved charges* Q_k, $k \geqslant 0$, $[H, Q_k] = 0$, (note that $Q_0 = 2H$)

$$Q_k = \sum_{m=-m_-}^{m_+} [h_m(U_{k+m} + U_{-k+m}) + g_m(V_{k+m} + V_{-k+m})]. \tag{16}$$

(ii) In another case, the solutions $a_\pm(\lambda), b_\pm(\lambda)$ are given by (11) and aready solve (7) globally (so $N = 1$). The logarithmic transfer matrix is now rather trivial, $T(\lambda) = \sum_{m=1}^{\infty}(V_m + V_{-m})\lambda^m$, giving the *conserved currents* C_k, $k \geqslant 0$, $[H, C_k] = 0$,

$$C_k = V_{k+1} + V_{-k-1} = Z_{[1(3^k)2]} - Z_{[2(3^k)1]}. \tag{17}$$

C_0 is the particle current of the associated spinless fermion model (via the Wigner–Jordan transformation), C_1 is the energy current, etc.

It can easily be verified directly such that $[T_{(i)}(\boldsymbol{\lambda}), T_{(i)}(\boldsymbol{\mu})] \equiv 0$, $[T_{(i)}(\boldsymbol{\lambda}), T_{(ii)}(\mu)] \equiv 0$, and $[T_{(ii)}(\lambda), T_{(ii)}(\mu)] \equiv 0$. Hence all the conservation laws are in involution $[Q_k, Q_l] = [Q_k, C_l] = [C_k, C_l] = 0$. For example, for the Ising model in a transversal magnetic field, $H = JU_1 + hU_0$, one recovers well known conservation laws $Q_k = J(U_{k+1} + U_{1-k}) + h(U_k + U_{-k})$ and C_k (17) which required more involved methods in [4].

Conservation laws in the non-autonomous case, kicked-Ising model

Next we study more general and possibly *non-autonomous* quantum spin chains which are propagated by members of a unitary Lie group generated by DLA $\mathfrak{S}$ which in general *cannot* be written in terms of some Hamiltonian H, as $\exp(-\mathrm{i}H)$. For simplicity, we consider *periodically kicked systems* which correspond to time-dependent Hamiltonian

$$H(t) = H_0 + \delta_p(t)H_1 \tag{18}$$

where $\delta_p(t)$ is a periodic delta function of period 1, and $H_0, H_1 \in \mathfrak{S}$ are the generators—the kinetic energy and the potential, respectively. Using the adjoint representation of DLA, the (linear) Heisenberg map U^{ad} of an observable $A \in \mathfrak{S}$ for one timestep is factorized as

$$A(t+1) = U^{\mathrm{ad}}A(t) = U_1^{\mathrm{ad}}U_0^{\mathrm{ad}}A(t) \tag{19}$$

where $U_p^{\mathrm{ad}}A = \exp(\mathrm{i}\,\mathrm{ad}\,H_p)A = \exp(\mathrm{i}H_p)A\exp(-\mathrm{i}H_p)$, is the propagation by the kinetic energy and the potential, for $p = 0, 1$, respectively. The transfer matrix is now sought by the invariance condition

$$U^{\mathrm{ad}}T(\boldsymbol{\lambda}) = T(\boldsymbol{\lambda}) \tag{20}$$

in the form (6). The method of the solution is analogous to (7)–(15) whereas the difference equations for a_n, b_n are now obtained by means of adjoint representation of propagators which can be derived explicitly by means of equations (3) and series expansion of the exponential function; for example if it is generated by U_m

$$\exp(\mathrm{i}\alpha\,\mathrm{ad}\,U_m)U_n = c^2U_n + s^2U_{2m-n} + cs(V_{n-m} - V_{m-n})$$
$$\exp(\mathrm{i}\alpha\,\mathrm{ad}\,U_m)V_n = c^2V_n + s^2V_{-n} - cs(U_{m+n} - U_{m-n})$$

where $c = \cos(2\alpha)$, $s = \sin(2\alpha)$.

Here, the general procedure cannot be written as explicitly as in the autonomous case, so we work out in detail an example of a *kicked-Ising* model where the kinetic generator is at the usual one-dimensional Ising Hamiltonian, $H_0 = JU_1 = \sum_j J\sigma_j^x\sigma_{j+1}^x$, and the kick potential is the transversal magnetic field, $H_1 = hU_0 = \sum_j h\sigma_j^z$. Condition (20) results in the system of second-order difference equations for a_n, a_{-n}, b_n, b_{-n} which is solved through the ansatz (8) giving the solution (again for any $|\lambda| < 1$)

$$\begin{aligned} a_+(\lambda) &= s_J c_h + c_J s_h \lambda^{-1} \qquad & b_+(\lambda) &= s_J s_h(\lambda - \lambda^{-1})/4 \\ a_-(\lambda) &= s_J c_h + c_J s_h \lambda \qquad & b_-(\lambda) &= -b_+(\lambda) \end{aligned} \tag{21}$$

where $s_J = \sin(2J)$, $c_J = \cos(2J)$, $s_h = \sin(2h)$, $c_h = \cos(2h)$, and the trivial solution (11). In order to obtain the global solution (a_n, b_n) we again glue together a linear combination of partial solutions for positive and negative n at the two points (since the system is of second order), for example at $n = 0, 1$. We obtain the system (14) for the three coefficients c_m, $N = 3$, depending on a triple of spectral parameters $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ with the solution (15). Collecting the terms with different powers of λ_m in the power series expansion of the logarithmic transfer matrix $T(\boldsymbol{\lambda})$ we obtain two infinite sets of conservation laws, namely the charges Q_k, $k \geqslant 0$, $U^{\mathrm{ad}} Q_k = Q_k$,

$$Q_k = c_J s_h (U_{k+1} + U_{-k+1}) + s_J c_h (U_k + U_{-k}) - s_J s_h (V_{k+1} + V_{-k+1} - V_{k-1} - V_{-k-1}) \tag{22}$$

and currents (17), C_k, $k \geqslant 0$, $U^{\mathrm{ad}} C_k = C_k$. The conservation laws of the kicked-Ising model (22) are identical to the invariants (16) of an autonomous IQM system with the Hamiltonian $H_{\mathrm{KI}} = Q_0/2 = c_J s_h U_1 + s_J c_h U_0 - s_J s_h (V_1 - V_{-1})/2$. Note, however, that the full dynamics are not identical, $\exp(\mathrm{i}\,\mathrm{ad}\, H_{\mathrm{KI}}) \neq U^{\mathrm{ad}}$.

Structure of DLA and the diagonalization of conserved charges

Let us now analyse the structure of DLA $\mathfrak{S}$ more carefully. The current invariants C_k are rather trivial; they span a *maximal ideal* $\mathfrak{I}$ of DLA $\mathfrak{S}$, $[\mathfrak{S}, \mathfrak{I}] = 0$. The derived (semisimple) DLA $\mathfrak{S}' = [\mathfrak{S}, \mathfrak{S}] = (\mathrm{ad}\,\mathfrak{S})^\infty \mathfrak{S} = \mathfrak{S}/\mathfrak{I}$ is spanned by $U_m \pm U_{-m}$ and $V_m - V_{-m}$, for $m \geqslant 0$, or in terms of *real non-local* Fourier transformed basis

$$\begin{aligned} J^1(\kappa) &= \frac{\mathrm{i}}{8\pi} \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\kappa m} (U_m - U_{-m}) \\ J^2(\kappa) &= \frac{\mathrm{i}}{8\pi} \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\kappa m} (V_m - V_{-m}) \\ J^3(\kappa) &= -\frac{1}{8\pi} \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\kappa m} (U_m + U_{-m}) \end{aligned} \tag{23}$$

for $0 \leqslant \kappa < \pi$, where the commutation relations read

$$[J^p(\kappa), J^r(\kappa')] = \mathrm{i}\delta(\kappa - \kappa') \sum_s \epsilon_{prs} J^s(\kappa). \tag{24}$$

Therefore, derived DLA is isomorphic to an infinite direct sum $\mathfrak{S}' \sim \bigoplus_{n=1}^{\infty} \mathfrak{su}_2$. It has an infinite rank, Cartan subalgebra is spanned by a continuous root basis $J^3(\kappa)$ and Chevalley generators are $J^{\pm}(\kappa) = J^1(\kappa) \pm \mathrm{i}J^2(\kappa)$. Now we construct the *vacuum state* $|\emptyset\rangle$ by the condition $J^-(\kappa)|\emptyset\rangle \equiv 0$, which is equivalent to $(U_m - U_{-m} - \mathrm{i}V_m + \mathrm{i}V_{-m})|\emptyset\rangle \equiv 0$, and also $\sigma_j^-|\emptyset\rangle \equiv 0$. Hence the vacuum is the state with *all spins down*. Let us write the Fourier transform of the currents and charges as, $C(\kappa) = \sum_n \exp(\mathrm{i}\kappa m) C_m$ and $Q(\kappa) = \sum_m \exp(\mathrm{i}\kappa m) Q_m = Q(-\kappa)$, respectively. Note that $Q(\kappa) = Q(-\kappa)$ since $Q_m = Q_{-m}$. Using the explicit form (16) we compute

$$\begin{aligned} Q(\kappa) &= -8\pi \boldsymbol{q}(\kappa) \cdot \boldsymbol{J}(\kappa) + g_r(\kappa) C(\kappa) \\ \boldsymbol{q}(\kappa) &= (h_i(\kappa), g_i(\kappa), h_r(\kappa)) \end{aligned} \tag{25}$$

where $h_r(\kappa) = \operatorname{Re} h(\exp(\mathrm{i}\kappa))$, $h_i(\kappa) = \operatorname{Im} h(\exp(\mathrm{i}\kappa))$, $g_r(\kappa) = \operatorname{Re} g(\exp(\mathrm{i}\kappa))$, $g_i(\kappa) = \operatorname{Im} g(\exp(\mathrm{i}\kappa))$. The structure (24) is invariant with respect to arbitrary local (κ-dependent) rotation (non-Abelian gauge transformation) of the vector field $\boldsymbol{J}(\kappa)$. Particularly interesting is the rotation $R(\kappa)$ around axis $\boldsymbol{a}(\kappa)$,

$$\boldsymbol{a}(\kappa) = \frac{(g_i(\kappa), -h_i(\kappa), 0)}{\sqrt{g_i^2(\kappa) + h_i^2(\kappa)}} \qquad \boldsymbol{a}(\kappa) \cdot \boldsymbol{q}(\kappa) \equiv 0$$

for an angle $\varphi(\kappa)$,

$$\varphi(\kappa) = \arctan \frac{\sqrt{g_i^2(\kappa) + h_i^2(\kappa)}}{h_r(\kappa)}$$

namely, $R\boldsymbol{r} = (\boldsymbol{a} \cdot \boldsymbol{r})\boldsymbol{a} + \frac{\boldsymbol{q}\cdot\boldsymbol{r}}{|\boldsymbol{q}|}\boldsymbol{k} + \frac{(\boldsymbol{a}\times\boldsymbol{q})\cdot\boldsymbol{r}}{|\boldsymbol{a}\times\boldsymbol{q}|}\frac{\boldsymbol{a}\times\boldsymbol{k}}{|\boldsymbol{a}\times\boldsymbol{k}|}$, where $\boldsymbol{k} = (0, 0, 1)$, which has the property $R(\kappa)\boldsymbol{q}(\kappa) = |\boldsymbol{q}(\kappa)|\boldsymbol{k}$. The unitary transformation of the vector field

$$\boldsymbol{W}(\kappa) = R(\kappa)\boldsymbol{J}(\kappa) = \exp\left(\mathrm{i}\int_0^\pi \mathrm{d}\kappa\, \varphi \boldsymbol{a} \cdot \boldsymbol{J}\right) \boldsymbol{J} \exp\left(-\mathrm{i}\int_0^\pi \mathrm{d}\kappa\, \varphi \boldsymbol{a} \cdot \boldsymbol{J}\right)$$

makes the the conserved charges $Q(\kappa)$ proportional to the new root basis $W_3(\kappa) = \boldsymbol{q}(\kappa) \cdot \boldsymbol{J}(\kappa)/|\boldsymbol{q}(\kappa)|$, namely

$$Q(\kappa) = -8\pi |\boldsymbol{q}(\kappa)| W^3(\kappa) + g_r(\kappa) C(\kappa). \tag{26}$$

Using the same rotation we construct a new vacuum state $|\emptyset\rangle_W$ relative to the field $\boldsymbol{W}(\kappa)$, $W^-(\kappa)|\emptyset\rangle_W \equiv 0$, namely,

$$|\emptyset\rangle_W = \exp\left(\mathrm{i}\int_0^\pi \mathrm{d}\kappa\, \varphi(\kappa)\boldsymbol{a}(\kappa) \cdot \boldsymbol{J}(\kappa)\right)|\emptyset\rangle.$$

Let us now discretize the momentum κ to L bins which corresponds to (but is not identical to) a finite chain of L spins, and define

$$W_k^p := \int_{\pi(k-1)/L}^{\pi k/L} \mathrm{d}\kappa\, W^p(\kappa), \qquad 1 \leqslant k \leqslant L.$$

Then we have $[W_k^p, W_l^r] = \mathrm{i}\delta_{kl} \sum_s \epsilon_{prs} W_k^s$. The eigenstates of conserved charges $Q(\kappa)$ (and of root basis W_k^3 since $[Q(\kappa), W^3(\kappa')] \equiv 0$) can be labelled by L binary quantum numbers $c_k \in \{0, 1\}$, $k = 1, \ldots, L$, and are constructed by means of creation operators

$$|c_k\rangle = \left(\prod_{1\leqslant k\leqslant L}^{c_k=1} W_k^+\right)|\emptyset\rangle_W \tag{27}$$

with $W_l^3|c_k\rangle = (c_l - \frac{1}{2})|c_k\rangle$. Hence, for smooth $|\boldsymbol{q}(\kappa)|$ and large L, all the charges (26) are *diagonal* in the eigenbasis (27). Of course, the *eigenvalues* are finite (for the infinite system $L = \infty$) only for the charge *densities* $Q'_m = \lim_{L\to\infty}(1/L) Q_m|_L$ and not for the *extensive* charges Q_m. The eigenvalues of invariant densities are computed by taking the limit $L \to \infty$ and the inverse cosine transform

$$Q'_m|c(\kappa)\rangle = -\frac{8}{\pi}\int_0^\pi \mathrm{d}\kappa'\, \cos(\kappa' m)|\boldsymbol{q}(\kappa')|[c(\kappa') - \tfrac{1}{2}]|c(\kappa)\rangle.$$

Label $c(\kappa)$ is an arbitrary (under certain restrictions, for example, being measurable) *index function* $c : [0, \pi) \to \{0, 1\}$ (for finite L, $c(\kappa) = c_k$, $\pi(k-1)/L \leqslant \kappa < \pi k/L$) and $|c(\kappa)\rangle$ is the corresponding eigenstate which should be properly defined by some limiting procedure $L \to \infty$ of (27). It seems that the spectrum of Q'_m is purely continuous. On the other hand, for the currents we have $C_m|c(\kappa)\rangle \equiv 0$, since $[C(\kappa), \boldsymbol{W}_l] \equiv 0$. Having such a transparent structure (24)–(27) it should be an easy task to compute physically interesting *correlation functions*.

In this letter we have introduced an *infinitely dimensional space* of completely IQM systems (spin-$\frac{1}{2}$ chains or chains of spinless fermions), the so-called DLA, as opposed to few-parameter families of completely IQM systems known in the literature to date. The model is an infinite-dimensional extension of the Ising model in transversal field (equivalent to the XY-model [4] and to the one-dimensional free fermion theory). For every element of the algebra which is interpreted as a Hamiltonian, or any propagator from the associated unitary Lie group being generated by a finite number of elements of the algebra (such as the Ising model periodically kicked by the transversal magnetic field), we construct two infinite sets of quantum invariants of motion, the conserved charges and the conserved currents. Is is shown heuristically how to diagonalize these conservation laws. Explicit expressions of the conserved charges are quite simple (much simpler than in general Heisenberg (XYZ) or the Hubbard model [5], for example) though non-trivial.

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